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A new method for single pile settlement prediction and analysis

W. G. K. FLEMING*

A method is presented for the analysis and prediction of single pile behaviour under maintained loading, based on the use of hyperbolic functions to describe individual shaft and base performance. When these functions are combined, and elastic pile shortening is added by a relatively simple procedure, an accurate model is obtained. By a simple method of linkage, which relies on the fact that a hyperbolic function of the type described requires only definition of its origin, its asymptote and either its initial slope or a single point on the function, conventional 'elastic' soil parameters and ultimate loads may be used to describe total performance. By means of the changing slope of such functions, this method also reflects well in the increase of soil moduli at low strains. Examples are given from back-analysis of some fully instrumented and other cast-in-place pile test results, to demonstrate that good agreement with all recorded features can be achieved using the model. Extensive use has confirmed its validity for maintained load tests in a wide range of soils. Provided that piles have been made to settle sufficiently under load, so that the latter part of each relationship is well defined beyond the stage where shaft friction is close to a constant value, all the main relevant parameters can be determined with good accuracy in back-analysis. The derived data may then be used to predict behaviour of piles in similar circumstances on other sites or of piles of different diameter in the same soils. Subject to the conditions described in the Paper, the method has far-reaching implications for design, construction and testing techniques.

L'article présente une méthode pour analyser et prédire le comportement d'un pieu unique sous chargement continu. Elle est basée sur l'emploi de fonctions hyperboliques pour décrire les performances du fût isolé et de la pointe. Lorsque ces fonctions sont combinées et qu'on y ajoute le raccourcissement élastique du pieu il en résulte un modèle précis. Par une méthode très simple de connexion il est possible d'employer des paramètres élastiques conventionnels du sol et des charges limites de rupture pour décrire les performances totales. Cette méthode reflète bien l'accroissement des modules du sol avec basses contraintes. Des examples sont présentés pour démontrer que l'emploi du modèle s'accorde bien avec toutes les données enregistrées. Son emploi frequent a confirmé sa validité pour des essais à chargements continus pour une large gamme de sols. Pourvu que les pieux soient assez enfoncés sous chargement, on trouve que tous les paramètres principaux importants peuvent être détermines avec une précision satisfaisante par analyse rétrospective. Alors il est possible d'employer les données dérivées pour prédire le comportement des pieux sous des circonstances analogues à d'autres emplacements ou bien de pieux de diamètre différent dans les mêmes sols. Cette méthode a des implications d'une grande portée pour les études, la construction et la technique des essais.

KEYWORDS: **analysis; bearing capacity; field tests; foundations; piles; settlement.**

INTRODUCTION

In his Rankine Lecture, Poulos (1989) catalogued the available methods for predicting pile performance under load, ranging from simple to complex methods using finite element solutions. He drew attention to the versatility of some of the more complex methods, but also demonstrated that in the realm of pile performance prediction, the result is only as good as the input information. The sophisticated input data required are not normally available from conventional site investigation, and there would therefore seem to be a place for a simpler approach that could readily be correlated with site experience and mainly used parameters that most geotechnical engineers would recognize and understand.

Chin (1970, 1972, 1983) has made the method of plotting the behaviour of both footings and piles according to the hyperbolic method well-known. This method has been widely adopted, although it has not been linked with soil parameters, but rather used as a method for defining ultimate loads.

Discussion on this Paper closes 4 January 1993; for further details see p. ii.

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Fellenius (1980) has discussed the Chin method and other methods for defining ultimate loads; he and others have drawn attention to the fact that the Chin method appears to overpredict. However, there is little doubt that in most cases, according to the plotting method, linear functions represent pile performance very well.

The method is expressed by Chin (1970, 1972) in the form $\Delta/P = m\Delta + C_1$, where Δ is pile head settlement, P is applied load and C_1 is a constant. Thus if Δ/P is plotted against an abscissa of Δ, a linear plot is obtained and the inverse slope $1/m$ gives an asymptotic limiting value of P. This, according to the evidence presented by Chin, is true of piles that carry most of their load by shaft friction, and also of footings and piles that carry most of their load in end bearing. A typical relationship between pile head settlement Δ and settlement divided by load Δ/P is shown in Fig. 1.

Many such relationships for piles are bilinear: it has been suggested by Chin & Vail and has often been accepted that the first part (A) of the relationship represents shaft friction while the second part (B) represents total load. This cannot be strictly true because of the nature of hyperbolic functions, but it can easily be accepted that individually shaft and base performance are of hyperbolic form.

It is interesting to speculate as to why the simple hyperbolic function should be important in the matter of foundation settlement. Chin (1983) suggests that mobilization of stress in a soil with increase of strain is a function of an increasing number of effective soil contacts rather than of a general increase of intergranular stress on a constant number of grain contacts. He suggests that intergranular stress in a flocculated clay, for example, is virtually constant and independent of the applied or effective stress. On this basis he derives a hyperbolic function for the stress–stain relationship. It may be visualized that when a soil is under compressive stress, the load is transmitted by internal columnar grain structures and that as these reach limiting loads, more and more columns begin to support load, each having approximately the same yield load. This is an interesting hypothesis, and appears plausible.

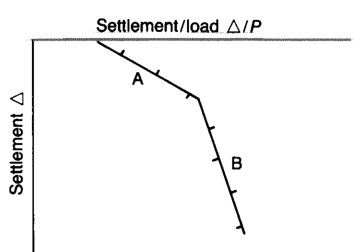

Fig. 1. Relationship of settlement and settlement/load

In this Paper a means of analysis and forecasting pile settlement based on the simple hyperbolic function is developed. It is first necessary to consider the obvious criticisms of the use of Chin's method in practice so that items that affect performance and are not normally hyperbolic can be separated from the general soil functions.

Two obvious features lead to the criticism that the method overpredicts ultimate load. First, by the nature of the function, the slope of the plotted lines represents an asymptote in each case. Most definitions of ultimate load are arbitrary, as Fellenius (1980) shows, being based either on a settlement related in some way to diameter or on geometrical manipulation. Most theoretically satisfactory bearing capacity coefficients are based on soil mechanisms that would automatically imply asymptotic values. However, asymptotic load values will always exceed those determined arbitrarily. The second distorting influence is the elastic shortening of the pile body, as can easily be demonstrated by making realistic estimates of shortening and removing this item from the settlement before plotting the functions.

It must also be borne in mind, that some driven piles, in particular, show the characteristic of set-up, which means that after installation their frictional capacity increases and on subsequent loading it declines at large strains. This may also be true of certain piles in soft sensitive alluvial deposits, but there is little evidence of it in cast in place piles in overconsolidated soils at least up to movements of the order of 5% of pile diameter. Within this range the stated hyperbolic function appears to hold true. Interestingly, Burland & Twine (1988) suggest that residual strengths apply along cast in place pile shaft surfaces in clay, and that under maintained loading conditions there is no decline in load following a peak value, this being a feature of a dynamic context, for example in CRP tests.

SETTLEMENT PREDICTION

Settlement and differential settlement are perhaps the most important features in pile design, and the problem is complicated by structural stiffness, pile load redistribution, construction techniques and group effects. Settlement control, however, receives the most attention and, if the performance of a single pile cannot be adequately forecast, it poses something of a dilemma as many specifications include numbers with which it is difficult to comply without some understanding of the mechanisms involved. Fortunately, most specifications are not concerned with group settlements, although the calculation methods based on elastic theory are of considerable help in this context. The use of empirical group load reduction factors is now generally discredited, as they have no basis apart from geometrical manipulation.

Cementation Piling Foundations Ltd has for many years addressed the problem of pile settlement by simple means, particularly for cases in which the design load of a pile exceeds the ultimate shaft friction, using the methods set out by Fleming & Thorburn (1983). It would be useful, however, to develop a function that could characterize pile load/settlement behaviour, and the hyperbolic function offers a key to this. Given that piles in general behave according to such a function with respect to shaft friction and end bearing, a method can be derived relatively simply by relating the performance to a continuous function which can mostly be linked to conventional soil parameters.

DEVELOPMENT OF BEHAVIOURAL MODEL

In Fig. 2, which represents a typical plot as used by Chin in considering a truly rigid pile, the slope of A represents ultimate shaft friction and the slope of B is the ultimate end bearing as defined by vertical asymptotes to the load/settlement relationships. Thus ultimate shaft friction is given by

$$U_S = \frac{\Delta_S}{(\Delta_S/P_S) - K_S} \tag{1}$$

where Δ_S represents settlement of the shaft head at any load P_S, and K_S is the intercept on the horizontal axis. Equation (1) can be rearranged to give

$$\Delta_S = \frac{K_S U_S P_S}{U_S - P_S} \tag{2}$$

Similarly, base performance can be expressed as

$$\Delta_B = \frac{K_B U_B P_B}{U_B - P_B} \tag{3}$$

where the load P_B corresponds to a settlement Δ_B. For a rigid pile, Δ_B is movement at the pile head.

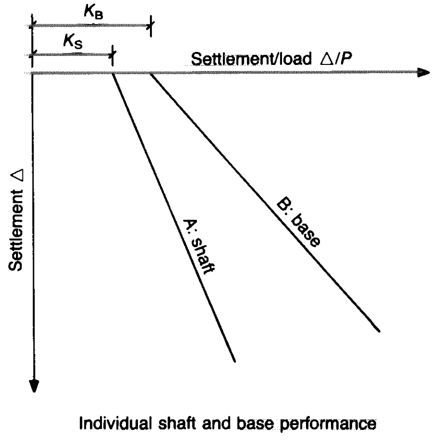

Fig. 2. Individual shaft and base performance

Shaft friction and settlement

There is substantial evidence that the settlement of a pile shaft for a given load is a direct function of the diameter D_S (see for example the finite element studies carried out by Randolph & Wroth (1982)). Similarly, a considerable number of studies seem to indicate that K_S is an inverse function of U_S, i.e. settlement for a given load decreases with increasing ultimate shaft load. Thus, from

$$K_S = \frac{M_S D_S}{U_S} \tag{4}$$

it is found that M_S becomes a dimensionless flexibility factor in the nature of an angular rotation, and equation (1) can be rewritten as

$$\Delta_S = \frac{M_S D_S P_S}{U_S - P_S} \tag{5}$$

M_S is in fact the tangent slope at the origin of the hyperbolic function representing shaft friction.

Randolph (1991) points out that M_S is the equivalent of $\zeta\tau_S/2G$ in the notation of Randolph and Wroth (1978, 1982) where ζ is $\ln(r_m/r_c)$, r_m is the radius at which soil deflexions become vanishingly small, r_c is the pile radius, τ_S is the shear stress at the pile surface and G is the soil shear modulus. M_S is also dimensionless in this notation. Because G/τ_S lies in the range 500–2000 in the findings of Randolph & Wroth, M_S would be expected to have values in the range 0·001–0·004.

Base load and settlement

As far as base performance is concerned, the settlement of a circular footing is commonly expressed as

$$\Delta_B = \frac{\pi}{4}\frac{q}{E_B} D_B(1 - \nu^2)f_1 \tag{6}$$

where E_B is the modulus of the soil under the footing, q is the applied base pressure, Δ_B is the base settlement, D_S is the diameter, ν is Poisson's ratio and f_1 is a standard settlement reduction factor related to foundation depth. For increasing load on a given foundation this means a linear relationship between load and settlement.

To evaluate the secant modulus E_B from a real load/settlement relationship in a standard way, it is usual to take its value at one quarter of the ultimate stress in non-linear functions. Thus in the case of piles equation (6) can be simplified to

$$\Delta_B = \frac{0 \cdot 6075 q D_B}{E_B} \tag{7}$$

by attributing values of, say, $\nu = 0{\cdot}3$ and $f_1 = 0{\cdot}85$.

If at a load of $U_B/4$, equations (3) and (7) are set equal, the coefficient K_B can be determined for the point where the hyperbolic function and the linear elastic functions intersect. Thus

$$K_B = \frac{0 \cdot 58}{D_B E_B} \approx \frac{0 \cdot 6}{D_B E_B} \tag{8}$$

This value of K_B can now be used to determine the whole of the hyperbolic function. Equation (3) can therefore be rewritten as

$$\Delta_B = \frac{0 \cdot 6 U_B P_B}{D_B E_B (U_B - P_B)} \tag{9}$$

This allows an expression for the total load/settlement relationship to be formulated. Note that within a hyperbolic function of this type it is necessary only to define the origin, the asymptote and one point (e.g. the E_{25} point) in order to define the whole function. Of course, the secant modulus value in such a function is highest at the origin and falls linearly with increasing load, to zero at the asymptote; this accords with general experience of high E values at low strain.

TOTAL SETTLEMENT OF A RIGID PILE

If a pile is purely rigid, then obviously the loads taken by the shaft and base can be added to give a total load at any given settlement Δ_T

$$\Delta_S = \Delta_B = \Delta_T \tag{10}$$

and the total load is

$$P_T = P_B + P_S \tag{11}$$

The shaft load is available from equation (5), and can be written

$$P_S = \frac{U_S \Delta_S}{M_S D_S + \Delta_S} \tag{12}$$

and the base load is available from equation (9)

$$P_B = \frac{D_B E_B \Delta_B U_B}{0 \cdot 6 U_B + D_B E_B \Delta_B} \tag{13}$$

These terms may be expressed more simply and handled in a general form by writing the expression for total applied load at a given settlement and inserting the total pile head settlement value Δ_T

$$P_T = \frac{a \Delta_T}{c + \Delta_T} + \frac{b \Delta_T}{d + e \Delta_T} \tag{14}$$

where $a = U_S$, $b = D_B E_B U_B$, $c = M_S D_B$, $d = 0{\cdot}6 U_B$ and $e = D_B E_B$.

To solve for Δ_T given any specific value of P_T, equation (14) has to be rearranged in the form

$$(eP_T - ae - b)\Delta_T^2 + (dP_T + ecP_T - ad - bc)\Delta_T + cdP_T = 0 \tag{15}$$

If for convenience we let $eP_T - ae - b = f$, $dP_T + ecP_T - ad - bc = g$ and $cdP_T = h$, this yields the solution

$$\Delta_T = \frac{-g \pm \sqrt{(g^2 - 4fh)}}{2f} \tag{16}$$

Only the positive resulting value of Δ_T is used.

ELASTIC SHORTENING

The elastic shortening of a pile shaft under load is clearly additional to settlement calculated by the above method, and must depend on the relative development of load transfer between the pile and soil along its length, as well as on any free length or near friction-free length at the pile head, and on the load being transferred at the pile base. To work out the elastic shortening accurately would require a considerable knowledge of the load transfer flexibility M_S along the shaft, and would involve an iterative method, whereby the pile was divided into elements and compatibility of strains was studied at given levels. This would make for a somewhat cumbersome procedure, involving the complication of varying soil strata and thickness.

It is suggested that a simplified method can be used: a study of some piles in which elastic shortening has been measured indicates the following method to be sufficiently accurate for most purposes. The simplified method is indicated in Fig. 3, which considers shortening in three stages

(*a*) a free or low friction length extending to a distance L_0 from the pile head
(*b*) a length L_F over which friction is transferred
(*c*) the whole pile shortening as a column after the ultimate shaft friction has been reached.

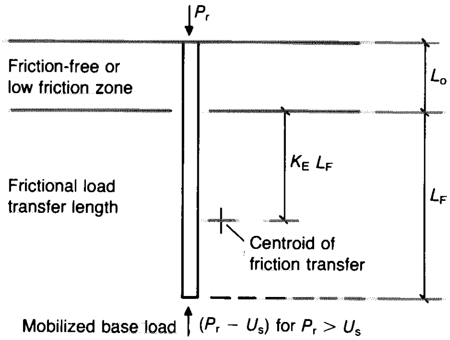

Fig. 3. Simplified method of calculating elastic shortening

The first of these elements is easily considered; the shortening Δ_1 is given by

$$\Delta_1 = \frac{4}{\pi} \frac{L_0 P_T}{D_S^2 E_C} \quad (17)$$

where E_C is Young's modulus for the pile material in compression.

The second stage represents the elastic shortening which takes place during load increase up to the stage when ultimate shaft friction has been mobilized. For uniform friction the elastic shortening will, at maximum, be equivalent to that of a column of length $0{\cdot}5L_F$. However, as M_S and K_E are both important elements in determining the early slope of the load/settlement relationship, and friction development takes place more rapidly at the top of this section than at its base, it seems preferable to use an effective column length slightly lower than $0{\cdot}5L_F$. A series of elastic shortening comparisons with the present method have been carried out using the Randolph and Wroth method in the form of the PIGLET computer program, based on an elastic soil of uniform stiffness. These indicate an effective column length for this case of $0{\cdot}4L_F$. Likewise, for a uniformly increasing soil stiffness and strength, from zero at the top of this length, full mobilization of friction would lead to an effective column length of $0{\cdot}67L_F$. The elastic method suggests that in the early stages of loading an effective length of $0{\cdot}47L_F$ is appropriate. For a typical London clay case, where strength increases linearly from a finite value at the top of the section, an equivalent column length of $0{\cdot}45L_F$ is a reasonable good approximation.

The effective column length appears to be between 70–80% of the distance from the top of the friction transfer length to the centroid of the friction load transfer diagram. If the coefficient applied to the friction length to give the effective column length is denoted as K_E, then shortening can be expressed as

$$\Delta_2 = \frac{4}{\pi} \frac{K_E L_F P_T}{D_S^2 E_C} \quad (18)$$

When the applied load P_T exceeds the ultimate shaft load U_S, additional load causes shortening of the full length L_F so that it may be treated simply as a column carrying the excess load, and the shortening of L_F becomes

$$\Delta_3 = \frac{4}{\pi} \frac{(P_T - U_S)L_F}{D_S^2 E_C} \quad (19)$$

As total elastic shortening Δ_E is the sum of the elemental shortenings being brought into play, for loads P_T up to the ultimate shaft load U_S

$$\Delta_E = \frac{4}{\pi} \frac{P_T(L_0 + K_E L_F)}{D_S^2 E_C} \quad (20)$$

and for greater loads

$$\Delta_E = \frac{4}{\pi} \frac{1}{D_S^2 E_C} [P_T(L_0 + L_F) - L_F U_S(1 - K_E)] \quad (21)$$

By the combination of equations (16) and (20) or (21) as appropriate, the total settlement of a pile for any load up to the ultimate load may be calculated, including a good estimate of elastic shortening.

A computer program has been written to facilitate rapid calculation, and given the name CEMSET. Help screens have been established to give guidance in the choice of numbers for various types of pile and soil.

APPLICATION OF THE METHOD

Having accepted that the hyperbolic function closely represents the load/settlement behaviour of piles, the method described is very simple, and its importance lies in its ability to link the function sensibly to well-recognized parameters. It suggests that one should use the asymptotic definition of failure instead of other arbitrary definitions which are confusing and difficult to interpret consistently. It also implies strongly that the application of any factor to ultimate load alone as a means of controlling deformation is crude and illogical.

The normalized plots for a wide range of soils, from the softest in which piles are likely to be used to very dense soils and soft rocks, are shown in Figs 4 and 5 for both shaft friction and end bearing as calculated. These show the familiar characteristics of pile load/settlement relationships in the rigid pile case. Note also that the shape and mathematical basis of the function

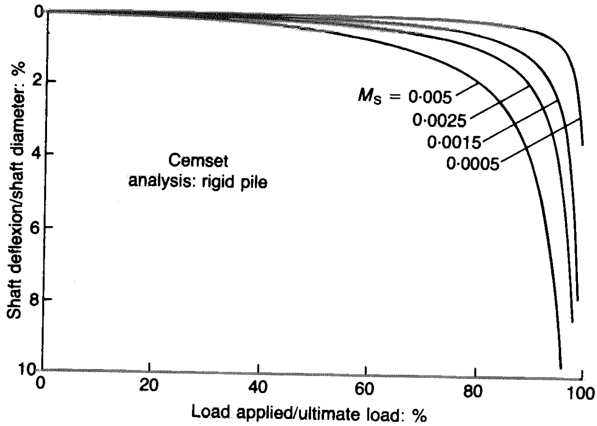

Fig. 4. Normalized plot of shaft friction settlement relationships for a range of soils from soft to very stiff (M_S = 0·005–0·0005)

imply that low strain moduli are always one-third higher than the E_{25} value.

It will be observed that, for example, the soil modulus below a pile base would be of the order of say 50 000 kN/m^2 for a stiff overconsolidated clay with an undrained shear strength approaching 200 kN/m^2, giving an E_B/q_u ratio of ~30, whereas the shaft flexibility factor M_S would be ~0·002. A simple comparison of the related curves shows that they are very different in character at this level of soil strength. This means that, if in a pile test the pile has been pushed sufficiently far to mobilize a reasonable part of the end bearing curve, the equations may be used to separate and back-calculate all the main parameters for the pile. As the base reaction stiffens and end bearing becomes more 'brittle', there is a remote possibility that the shaft and base characteristics may become too similar to separate mathematically in a reliable way. Only a very small proportion of the total range of piles are likely to be in this category.

If Figs 4 and 5 are used to judge pile performance without reference to the formulae, care should be taken that the scales are similar to those used in the diagrams. The graphs, however, are fully dimensionless and general given the conditions attached to equation (7).

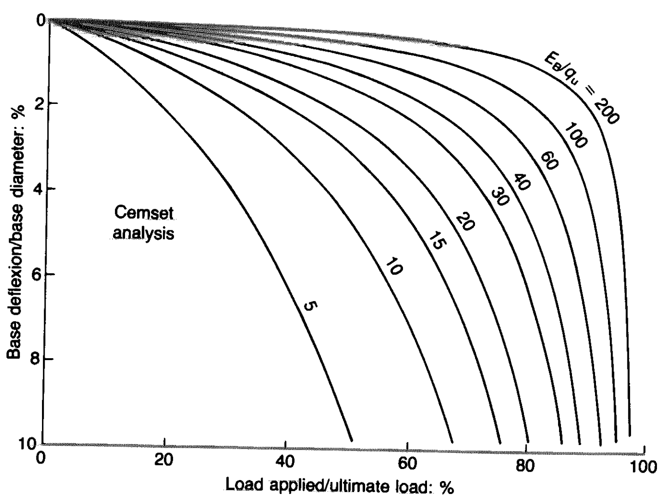

Fig. 5. Normalized plot of end bearing/settlement relationships for a range of soils from soft to very stiff: q_u = ultimate base pressure (kN/m^2); E_B = modulus of soil below base; E_B/q_u = 5–200; the latter value corresponds to soft rock materials

It should be stressed that good quality test data are required for accurate mathematical separation during back-analysis. Tests carried out commercially are often good enough to give reasonable indications of the various parameters, but any improvement in test quality leads to much greater accuracy. Quality often suffers because of an inability to hold loads constant in maintained load tests, and because test procedures have long and short load holding periods which give inconsistent degrees of consolidation or creep at the various stages. It also appears that the longer loads are held constant at any stage, the greater are the errors in the measuring system as a result of factors such as temperature variation. For the purposes of accurate back-analysis, the settlement at each load hold should be projected to infinite time before plotting the points to be used. It is found that the results of continuous rate of penetration tests can be analysed approximately, but the stiffness coefficients obtained are naturally higher than one obtains from maintained load tests. It is also clear that, as Burland & Twine (1988) suggest, ultimate shaft load is increased with subsequent decline if this test procedure is followed, so that rather distorted ultimate load results are obtained.

The method described has been used both as a predictive and an analytical tool for maintained load tests by Cementation Piling Foundations Ltd for three years. It has proved useful in assessing whether or not piles under test will perform according to specification, and in discovering appropriate parameters to use in future designs. In the analysis mode it is analogous to the signal matching procedure now used in dynamic load testing, and it similarly requires a certain degree of movement to acquire adequate data. Its advantage in comparison with dynamic signal matching lies in the fact that the dynamic procedure involves many more parameters and at present relies on an inferior bilinear model. The following comments on the various parameters may be helpful.

Diameters

The diameters of the shaft and base are regarded as known items (D_S and D_B). Equivalent diameters can be used for non-circular sections.

Length

The overall length must be known. The component L_0 is the free length or length through fill or soft alluvial deposits from the pile head. These soft soils rarely contribute significantly to bearing capacity. The component L_f is the pile length transmitting load by shaft friction.

Effective column length factor K_E

This factor converts the length L_F to an effective free column length. It is necessary first to find the centroid of friction transfer by calculation. The friction length down to the centroid should be multiplied by a factor in the range 0·7–0·8. In stiff overconsolidated clays, which increase in stiffness with depth, K_E is usually ~0·45.

Shaft flexibility factor M_S

This is found to vary from 0·004 in soft to firm or relatively loose soils to ~0·0005 in very stiff soils or soft rocks. As stated, it lies in the range that would be expected from Randolph and Wroth (1978), and decreases with increasing soil stiffness. In stiff overconsolidated clays, for example, it is found to be in the range 0·001–0·002, although some variations are found, even on a single site, which appear to be related to pile type, construction practice, pile straightness and possibly time-dependent construction processes.

Modulus of soil beneath pile base E_B

Back-analysis shows this to be one of the most interesting parameters of the method. It is obviously related to the intrinsic soil properties, but it is also highly construction dependent. There is a wide range of choices, depending on whether a pile is driven or bored, and pile base condition is very important.

Overconsolidation has an important effect on most soils. As site investigations as carried out at present are more concerned in practice with strength than with deformation, this factor is not usually directly determinable. Instead, there are several attempts in the literature to establish stiffnesses by correlation with other soil properties, for example by Meigh (1987), Burland & Burbridge (1985) and Stroud (1989). These are helpful in regard to the factors that generally influence stiffness, but data from a pile loading test seem to be best, as they also incorporate the construction factors. Indeed, it would seem highly desirable to test piles to higher loads and greater settlements than is done at present in order to establish all the parameters reliably.

Concrete modulus E_C

In practical terms it seems highly desirable to obtain the E_C value directly from the material of the pile. A common figure for concrete piles at the age of test is ~30 × 10^6 kN/m^2, but with high strength mixes and excellent curing conditions in cast-in-place piles, values as high as 50 × 10^6 kN/m^2 and infrequently higher seem to occur. A short extensometer or set of extensometers in the head region of a pile, but outside the zone of stress concentration below the load application level, seems an adequate answer to the problem. An alternative might be to cast a short dummy pile nearby, which could be extracted and tested in a testing press concurrent with pile loading.

Ultimate shaft load U_S

At present, conventional means of calculation are used for forecasting the ultimate shaft load. However, back-analysis shows that in reality conventional calculation is usually conservative but occasionally not so, possibly due to installation techniques that may alter the surrounding soil properties. This is particularly likely to happen in the interglacial sands and silts with certain types of bored pile.

Ultimate base load U_B

The ultimate base load is also calculated by conventional means for the purpose of prediction. Again, using the logical asymptotic definition it seems from analysis that for deep bases in clay soils the N_C factor is slightly higher than the conventional value of 9. Installation method is of primary importance, particularly for short piles, which rely heavily on end bearing. For conventional bored piles in such circumstances the cleaning of bases is important, continuous flight auger piles behave well given good construction techniques, and driven piles obviously densify cohesionless soils markedly in most cases. The stiffness of the soil in such circumstances may be increased by a factor of two or three for a driven pile, and is often even higher where the technique of driving bulbs is used. Data on all the parameters are currently being collected for a wide range of pile types and ground conditions; it is hoped to publish the more important findings in due course.

Sensitivity

From the equations and Figs 4 and 5 it will readily be appreciated that the most important parameters in the early stages of any pile load settlement relationship are the M_S and E_C values. Fortunately, in most cases these parameters have very limited ranges and have only minor effects on the ultimate shaft friction, end bearing and base soil stiffness moduli where movements are large in back-analysis. The E_B and U_B values have significantly different effects, and with sufficient settlement data can be separated readily. The most important consideration is that if piles are made to settle well beyond the stage where shaft friction is fully mobilized, potential errors in all the parameter determinations are greatly diminished. Using the computer program, it is a simple matter to investigate sensitivity in any particular case, and it can easily be appreciated that sensitivity depends on the relative magnitude of the parameters in individual situations.

Examples

A large and growing number of field test results have been examined by this method, and it is clear that with good data piles in a wide range of soil conditions follow the calculated form very closely indeed. At present, most of the piles that have been examined are of the cast in place type. Where it is possible to find instrumented pile tests, the data are usually good enough to confirm that the base alone, the elastic shortening and the pile as a whole can be modelled closely.

The following examples have been selected from the database of pile tests back-analysed by the method to illustrate its application in a range of ground conditions and for cast in place piles of different types. It is a simple matter when the database is sufficiently large to use the method for prediction purposes, as the main parameters are remarkably consistent with specific ground conditions and installation. The database currently extends to some 200 cases. All the input data points used in the examples are taken directly from site records.

Bored piles in stiff clay soils

Useful information can be found in Whitaker & Cooke (1966), which deals with instrumented tests carried out at Wembley on both straight shafted rotary bored piles and under-reamed piles. The paper provides information on the soil conditions, and although the maintained load data are given in detail over only part of the total load settlement curve, the ultimate base and shaft loads are quite accurately known. Whitaker & Cooke took a definition of failure as corresponding to about 10% of pile base diameter, and the ultimate loads were determined by continuous penetration tests. The data are fuller in some cases than in others; the maintained load results for two straight shafted piles have been chosen for illustration purposes.

In each case the compatibility of the solution has been checked against the ultimate load given, and the base/settlement relationship has been checked independently. These piles were not made to settle sufficiently during the maintained load test to give a clear solution for the base from the overall settlement data, but, usefully, this is supplemented by records from each pile base, allowing a full solution to be obtained. The solu-

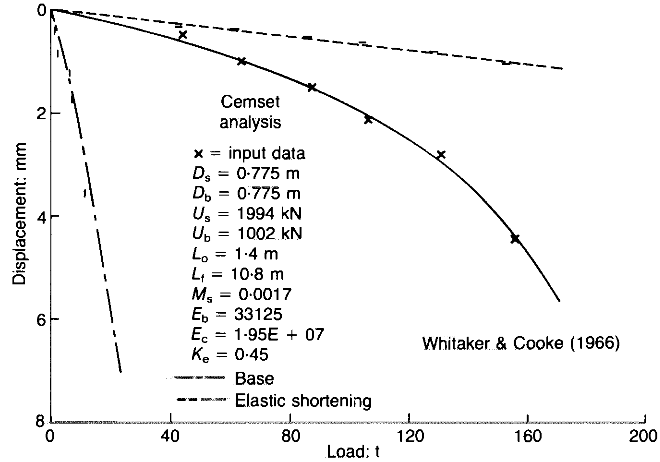

Fig. 6. Comparison of results from the present study with those of Whitaker & Cooke (1966): pile H

tion given in Figs 6 and 7 is entirely compatible with all the information supplied. For pile H, Whitaker & Cooke quoted an ultimate shaft load of 1960 kN and a base load of 770 kN. These figures correspond to asymptotic values of 1994 and 1009 kN respectively. For pile N, ultimate shaft and base loads were given as 3070 and 870 kN; the analysis by this method corresponds to asymptotic values of 3100 and 1068 kN. The analysis values correspond closely to the Whitaker and Cooke values, if 10% of diameter settlement criterion is taken as defining ultimate load. The method represents well the performance of bored piles in stiff overconsolidated clay.

Under-reamed pile in stiff clay

It is difficult to find results for under-reamed piles that have been made to settle significantly; again the work of Whitaker & Cooke (1966) at Wembley provides an interesting case. No satisfactory and straightforward result for the behaviour of under-reamed bases at this site could be found by the matching program until the original paper was studied more carefully. Pile P has been taken as an example. The under-reaming tool produced a dome-shaped upper surface, and did not at the time conform with usual specification requirements that the side slope should make an angle of 60° or more with the under-ream floor.

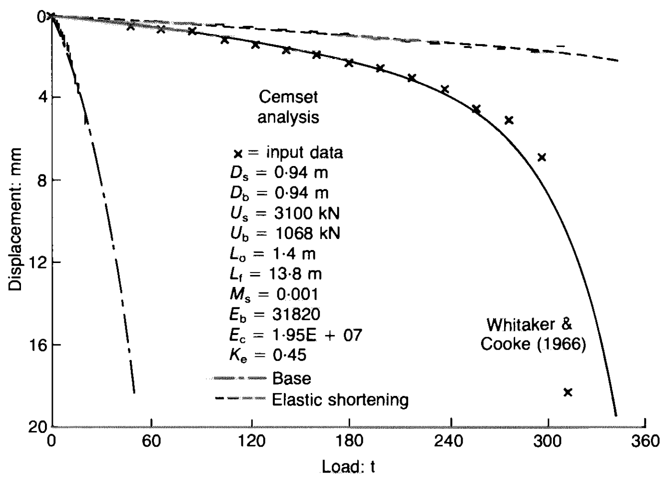

Fig. 7. Comparison of results from the present study with those of Whitaker & Cooke (1966): pile N

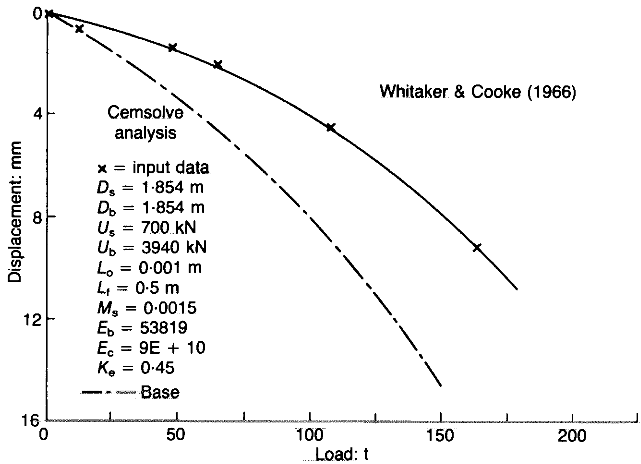

Fig. 8. Under-reamed pile P at Wembley: base characteristic only

Hence the under-ream was cut, then the shaft was deepened a little and a second cut was made. This allowed the composite under-ream to meet the specification requirement, but inevitably produced a peripheral surface on which friction could act. Once friction was allowed a part in base performance, the solution shown in Fig. 8 resulted for pile base capacity; the consequent solution for the complete pile is shown in Fig. 9. In the total solution, the 700 kN of shaft resistance on the base now appears in the shaft result, and the overall solution is exactly compatible with that given by Whitaker and Cooke, bearing in mind their criterion of approximately 10% of base diameter for the ultimate condition.

The method can thus represent well the case of an under-reamed large diameter bored pile in all its aspects in stiff overconsolidated clay, and can expose features of construction that might otherwise go unnoticed.

Driven piles in dense sand

The examples given in Figs 10 and 11 are from de Beer, Lousberg, de Jonghe, Wallays & Carpentier (1979). A series of Franki piles, with and without enlarged bases, were driven through ~8 m of very soft clay and peat to a penetration of just over 1 m in very dense sand. The enlarged based piles were subsequently extracted and measured, so the dimensions are fully known.

Although four of these piles have been examined in detail, two have been chosen to exemplify the results. Fig. 10 shows the results of analysis on pile V in the series, a straight-shafted pile. This pile was cast within a 406 mm steel tube: for the

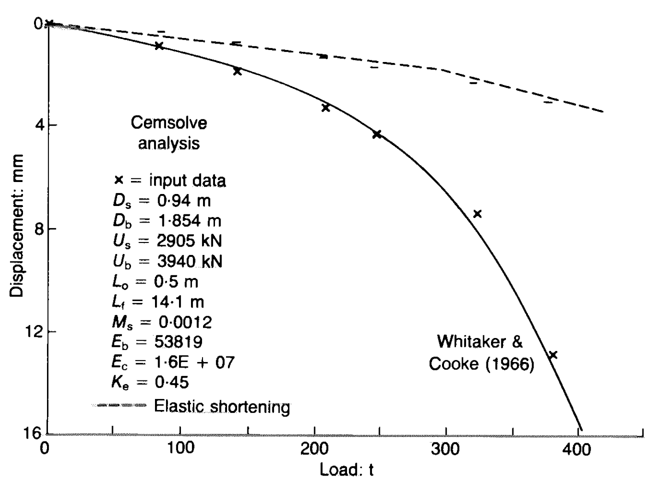

Fig. 9. Under-reamed pile P at Wembley: total pile performance

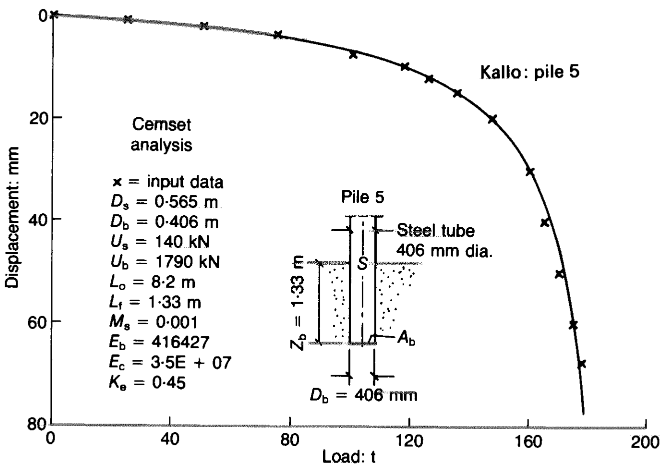

Fig. 10. Result of pile load test on a driven cast-in-place pile at Kallo, near Antwerp

purpose of calculating elastic shortening the steel area has been converted to equivalent concrete by the modular ratio method. The ultimate shaft load was very small and could not be determined accurately, but the base settlement characteristic clearly conforms with Fig. 5, the soil modulus E_B value being approximately 416 000 kN/m^2 and the ratio E_B/q_u being 30.

Pile 2 (Fig. 11) was driven through a slip-sleeve arrangement and had an enlarged base. The friction on this pile was effectively removed, and the soil modulus below the base now appears as 1000 000 kN/m^2. Again the form of the result is as indicated in Fig. 5 ($E_B/q_u = 79$). The method represents well the performance of driven piles in a dense sand both for straight-shafted piles and piles with enlarged bases. It is of interest to note that the stiffness of the base reaction is substantially different: this is much more likely to be due to construction technique than to natural variation in the founding layer.

Bored piles in chalk

Figure 12 shows the results of a test on a pile in chalk at Norwich. This was an instrumented pile, for which data have kindly been provided by Ove Arup. The chalk in this instance has standard penetration test results of the order of $N = 10$, and because the pile was instrumented by the Building Research Establishment, the base, total and elastic shortening characteristics were all

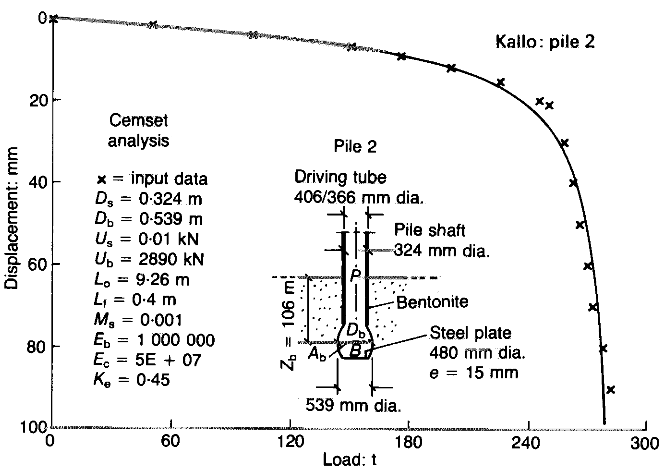

Fig. 11. Result of pile load test on a driven cast-in-place pile at Kallo, near Antwerp

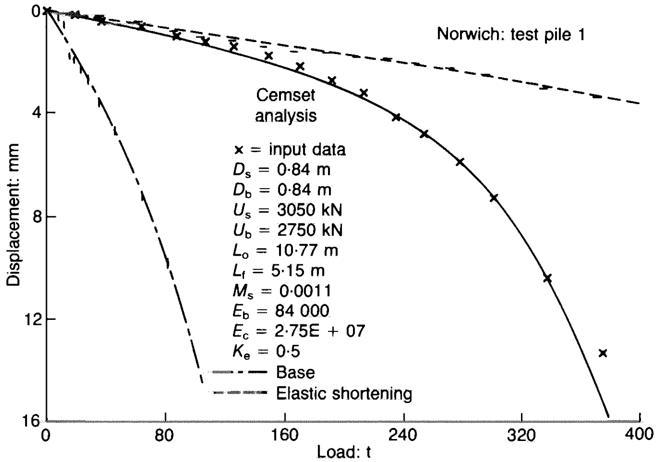

Fig. 12. Large diameter pile in soft chalk

measured. Fig. 12 shows the agreement of the solution with all the data. It will be observed that although the chalk was generally of relatively poor quality, the base soil modulus is 84000 kN/m^2, and while this value may be due to harder layers in the soft chalk, there is evidence that even soft chalks show relatively high modulus values. The details of this pile are in Twine & Grose (1989).

Again, the method represents well in detail the results of an instrumented large diameter pile in chalk. This chalk was weak according to the site investigation information, but the end bearing is higher and the base stiffness is greater than might have been expected. The elastic shortening is also well represented. Note that there are several piles tested in chalk in the database, with chalk ranging up to very hard, and that very good matching is possible in all cases.

Piles in silty conditions

The results of two tests on piles constructed using continuous flight augers at Shrewsbury in very complex silty conditions are shown in Figs 13 and 14. These piles are of interest because they were made to settle a long way under load. Below some 6 or 7 m of organic silt and clay there were layers of very silty sands, clayey silts and silty clays. Results of Dutch cone tests varied violently with friction ratios in the range 2–4. These results and those from standard penetration tests imply loose to at best medium-dense conditions, with SPT N values increasing from 8 or 9 at the top to

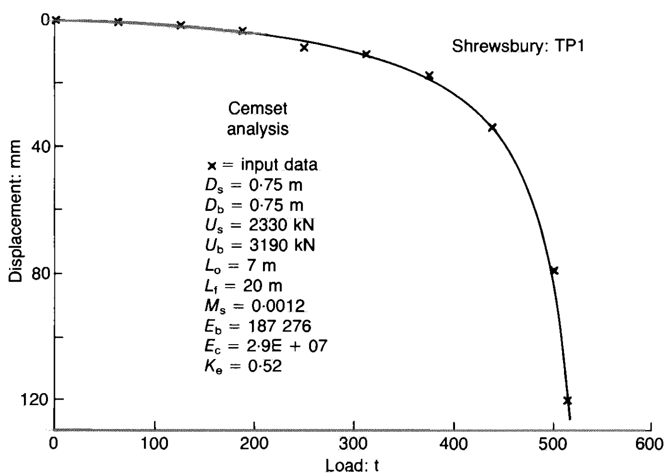

Fig. 13. 750 mm continuous flight auger pile 27 m long, in Shrewsbury

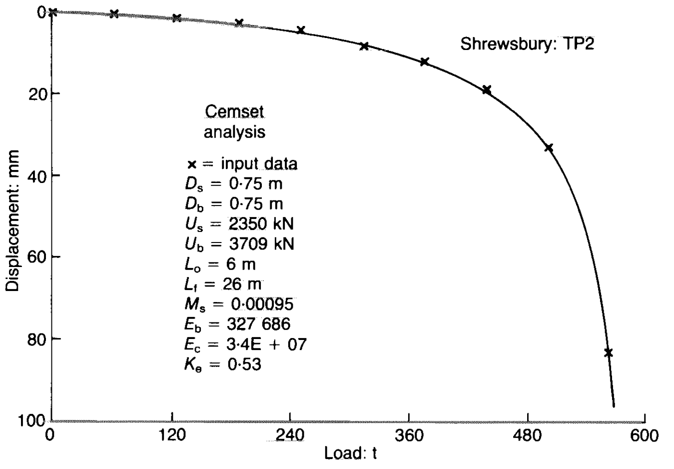

Fig. 14. 750 mm continuous flight auger pile, 32 m long, in Shrewsbury

about 20–30 near the pile bases. Groundwater stood near the piling surface level. Towards the lower end of each pile conditions became a little more sandy, but bands of silty clay and clayey silt persisted. The area is well known for its difficult piling conditions.

The results of test loading are of good quality and show that in spite of the very mixed ground conditions, end bearing is a more significant component of capacity than might have been expected. The full computer solutions are shown; again good agreements with the hyperbolic equation forms are apparent.

These are long piles, and the Young's modulus for the concrete may not be exact. If the concrete modulus is varied it is found to have only a very minor influence on the resulting load distribution between shaft and base, using the least squares curve fitting method contained within the analysis program for all the variable parameters. The matching and the values of parameters are good and stable, particularly in the case of test pile 2, where the data quality is better.

Pile in weathered Mercia mudstone

Figure 15 shows the results of a test on a continuous flight auger pile founded in a weathered Mercia mudstone in the Bristol area. Again the pile was made to deflect sufficiently to give a good fix on the various parameters. The soil was layered with softer and harder bands in the region of the toe of the pile, but it is evident that the pile base behaved in accordance with effective

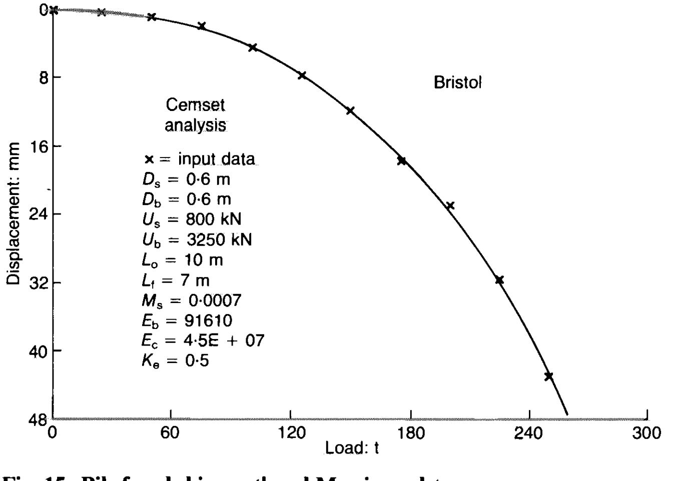

Fig. 15. Pile founded in weathered Mercia mudstone

parameters. Once the parameters are known, it is necessary only to insert the changes of pile diameter into the equations to discover the likely performance of contract piles, and whether or not they will comply with specification.

Internal pile instrumentation is expensive and there are many cases, such as continuous flight auger piles, in which full load-recording equipment cannot at present be inserted to positions where it would be useful. Subject to the conditions stated, this method would appear to offer valid results at the reasonably low cost of sufficient load application. Direct measurement of concrete elastic properties would be a useful and fairly straightforward addition to the system.

The method has been shown to have many consequences. The load/deformation performance of piles is not a matter of random behaviour.

NOTATION

- D_B diameter of pile base
- D_S diameter of pile shaft
- E_B deformation secant modulus for soil beneath pile base at 25% of ultimate stress
- E_C Young's modulus of pile concrete
- E_P Young's modulus for any pile material
- K_E effective column length of shaft transferring friction, divided by L_F
- K_S, K_B intercepts on settlement/load axis when settlement is plotted against settlement/load
- L_0 upper length of a pile carrying no load or low loads by friction
- L_F length of a pile transferring load to the soil by friction
- G shear modulus of soil
- M_S flexibility factor representing movement of a pile relative to the soil when transferring load by friction (dimensionless)
- N standard penetration test result
- P load applied at pile head
- P_B load applied at pile base
- P_S load applied to pile, carried by friction
- P_T load ($P_B + P_S$) applied at pile head
- U_S ultimate shaft friction load
- U_B ultimate pile base load
- a, b, c, d, e, f, g compound parameters
- C_1 constant (Chin model)
- C_U undrained shear strength of clay
- f_1 depth factor related to depth of foundation below ground
- m slope of line relating settlement to settlement/load (Chin model)
- q stress due to applied load at pile base
- r_c pile radius (Randolph model)
- r_m radius at which soil deflexions become vanishingly small (Randolph model)
- α adhesion factor
- Δ settlement
- Δ_B settlement of pile base under applied load
- Δ_E total elastic shortening of pile
- Δ_S settlement of pile shaft under applied load
- Δ_T total settlement of rigid pile under applied load P
- $\Delta_1, \Delta_2, \Delta_3$ components of elastic shortening of pile
- ζ $\ln (r_m/r_c)$
- ν Poisson's ratio
- τ_S shear stress at pile surface

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DISCUSSION

A new method for single pile settlement prediction and analysis

W. G. K. FLEMING (1992). *Géotechnique* **42**, No. 3, 411–425

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The state of stress in the soil surrounding a pile is complex. This is due to the process of installation and subsequent reconsolidation, and also to the stresses which existed in the soil prior to installation, which may not be known. As a result, when the pile is loaded the settlement is difficult to predict, and the level of sophistication which it is sensible to use in theoretical methods must be matched accordingly. The Author recognizes this. The complex problem of single pile settlement is globally characterized by a few parameters related to the hyperbolic law, and these are then found from the load–settlement curve. The numerous cases which the Author has successfully analysed give confidence in the method.

Perhaps the greatest objection to the hyperbolic law is the assumption that, irrespective of soil type or pile make-up a load–settlement curve when plotted on hyperbolic axes (namely settlement/load against settlement) should give a straight line. To define the hyperbola requires two parameters, and it is difficult to see how these relate to the engineering parameters of the pile and soil.

In the Paper this is partially overcome by the use of hyperbolic laws for both the shaft and the base. The four parameters are related to basic soil constants and the method as presented is a significant step forward in the understanding of pile behaviour. However, a consequence of using two hyperbolic relations is that the original assumption that load and settlement conform to a hyperbolic law is now violated. This is readily seen in the case of a rigid pile, given by equation (14), where the graph of Δ_T/P_T against Δ_T is no longer linear.

To overcome this problem, the Author suggests that the first part of the curve be associated with shaft parameters, while the second is associated with base parameters. The difficulty with this is knowing where shaft influence ends and base influence takes over. This can be seen in the case study of pile H at Wembley. Using the data in Fig. 6, and a standard non-linear structural programme, the load–settlement curve for this pile was obtained. This is shown on a hyperbolic plot in Fig. 16. Over the range of loading considered by the Author this gives a gentle curve with no apparent transition from shaft dominance to base dominance. This appears to conflict with the type of behaviour indicated by Fig. 1, and so I suggest that abrupt changes could be due to brittle behaviour in the soil.

The Paper gives persuasive arguments, mainly of a practical kind, for collecting together all the characteristics of shaft behaviour into a single hyperbolic relationship. I believe a better method is to represent shaft behaviour in terms of hyperbolic load transfer functions. This has been done in connection with the related problem of finding the form of dynamic load transfer functions which should be used in pile driving studies. The practical difficulties of taking dynamic measurements initially led me to study static load–settlement curves. Several factors arising from these studies have a direct counterpart in the present Paper.

The first concerns the law used for the basic load transfer function. In pile driving this is generally taken as bilinear (Fig. 17). The Author regards this as an inferior law, but it is necessary to remember that its use is necessitated by the practical need to develop simple numerical procedures for the unloading and reversed loading ranges. The bilinear law, like the hyperbola, requires two parameters to define it, one of which is the ultimate resistance. However, unlike the hyperbola, this is reached after a finite displacement Q, which is known as the quake. For the shaft, Q can be associated with $M_S D_S$; for the base its counterpart is $0{\cdot}6\ U_B/E_B D_B$ (see Table 1).

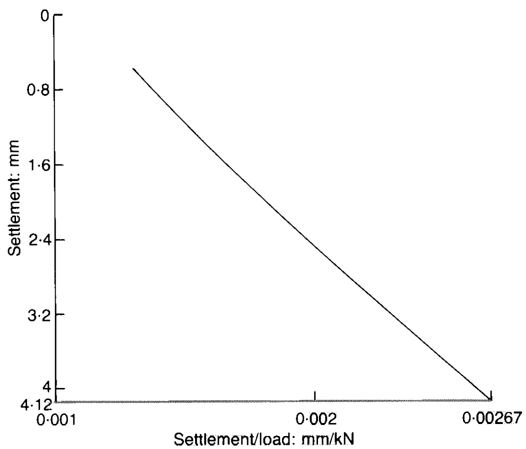

Fig. 16

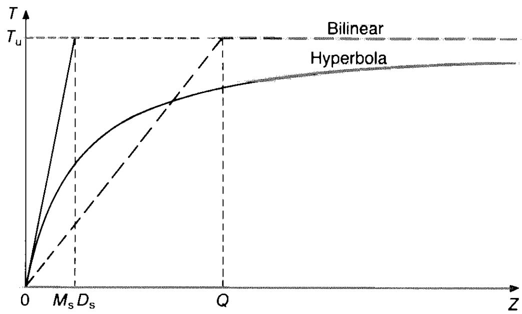

Fig. 17

The Author's comment that M_S does not appear to be sensitive to soil type is significant. This is also found to be the case with quake Q, which engineers often assume to be 2·5 mm.

The position with regard to the factor $0{\cdot}6\ U_B/E_B D_B$ is less well defined. Table 1 shows this to be more sensitive than $M_S D_S$ to soil type. I think that this is a consequence of the end resistance continuing to increase with displacement and therefore showing no tendency to approach a horizontal asymptote. This has implications for the values of end quake currently assumed in pile-driving analyses.

I should like to comment on the influence of shaft flexibility. The method devised by Poskitt & Ward (1988) has some similarities with that proposed by the Author, but proved to be ill-conditioned. As a result, the parameters obtained from back-analysis of field data were found to be sensitive to small changes in the data. From this I concluded that it was necessary to treat the pile and soil as a properly formulated non-linear structural system and solve it accordingly. By comparison, the Author's formulation appears to be well conditioned, and it is not immediately apparent why this should be so. Nevertheless, the efficacy of the method seems beyond doubt and the parameters obtained in the examples are acceptable whether they are interpreted for load testing or for the less usual application of quake in pile-driving calculations.

Table 1. Load test parameters

Site	$M_S D_S$: mm	$0{\cdot}6\ U_B/E_B D_B$: mm
Wembley H	1·32	23·4
Wembley N	0·94	21·4
Wembley P	1·13	23·7
Kallo 5	0·57	6·4
Kallo 2	0·32	3·2
Norwich 1	0·92	23·4
Shrewsbury TP1	0·90	13·6
Shrewsbury TP2	0·71	9·1
Bristol	0·42	35·5

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To evaluate non-linear single pile settlement, the Author uses the hyperbolic load-transfer function proposed by Chin (1970) which we used to present a computer code based on a pile finite element discretization which takes into account the non-linearity of the soil–pile interaction (Castelli, Maugeri & Motta, 1992). We also proposed a simplified procedure in a closed form similar to that proposed by the Author.

The computer code was used to back-analyse 12 loading tests of bored piles. Numerical analysis was carried out on instrumental full-scale pile tests, collected from existing literature. The piles were bored in clayey, silty, sandy and pyroclastic soils, ranged between 14 m and 42 m long and had diameters ranging between 0·42 m and 2 m. Assuming a hyperbolic load-transfer function, the back-analysis was performed with the aim of deducing the most appropriate values of the main parameters. The shaft flexibility factor can be evaluated using $M_S = 0{\cdot}001$–$0{\cdot}002$ when the unit ultimate skin friction is greater than 50 kPa and $M_S = 0{\cdot}002$–$0{\cdot}005$ when the unit ultimate skin friction is less than 50 kPa. These values are very close to those given by the Author when applying the Randolph and Wroth theory (1978).

The Author suggests evaluating the contribution of the settlement due to the elastic shortening by three stages Δ_1, Δ_2 and Δ_3. An alternative procedure could be to assume the soil to be homogeneous and to evaluate the elastic shortening in one stage, considering two laws of soil–pile interaction—along the pile shaft and at the base—with different stiffnesses. This gives the second order differential equation

$$y'' - \alpha^2 y = 0 \qquad (22)$$

where y is the settlement at depth z and α^2 is given by

$$\alpha^2 = 4K_L/E_c \pi D^2 \qquad (23)$$

where K_L is the stiffness of the lateral load-transfer function and D is the pile diameter. With the boundary conditions

$$E_c \frac{\pi D^2}{4} y'(0) = -P_T \qquad (24)$$

$$E_c \frac{\pi D^2}{4} y'(L) = -(P_T - P_S) = -K_p y(L) \qquad (25)$$

where K_P is the stiffness of the load-transfer function at the base and L is the total length of the pile, the following equation can then be derived for the elastic shortening

$$\Delta_e = C(4P_T/\alpha E_c \pi D^2) \quad (26)$$

where

$$C = \frac{e^{\alpha L}(1+\beta) + e^{-\alpha L}(1-\beta) - 2}{e^{\alpha L}(1+\beta) - e^{-\alpha L}(1-\beta)} \quad (27)$$

$$\beta = 4K_P/\alpha E_C \pi D^2 \quad (28)$$

To consider the non-linearity of the load–settlement curve, the value of K_L must be chosen depending on the load level η

$$K_L = Ki_L(1-\eta) \quad (29)$$

where Ki_L is the stiffness at the origin and

$$\eta = P_T/(U_S + U_B) \quad (30)$$

Thus this procedure, which takes into account both the shaft and base interactions as well as the load level, also shows the non-linearity of the elastic shortening.

Values of Ki_L may be derived as $Ki_L = U_S/M_S D$, and the following approximate relationship, deduced from parametric back-analysis, can be used to calculate K_P

$$(q_c/K_p D) = 0{\cdot}03 \quad (31)$$

where q_c is the point resistance deduced from static penetration testing.

Figures 18 and 19 compare the head settlements evaluated using the Author's procedure with ours for piles H and N (Whitaker & Cooke, 1966). Both procedures were also applied in one of the 12 loading tests (Viggiani & Vinale, 1983), which was used for the back-analysis, as shown in Fig. 20. All the results show good agreement between measured and computed settlements, irrespective of the procedure used.

For the Viggiani & Vinale (1983) pile, Fig. 21 shows the elastic shortening Δ_e derived from equations (26)–(31) compared with that derived from equations (17)–(21). Although our procedure predicts a non-linear elastic shortening and the Author's predict a linear one, the results are in good agreement. The main difficulty in applying these methods is in the correct determination of the function parameters. If it is possible therefore for the load-transfer function to be characterized accurately by using a simplified procedure, ana-

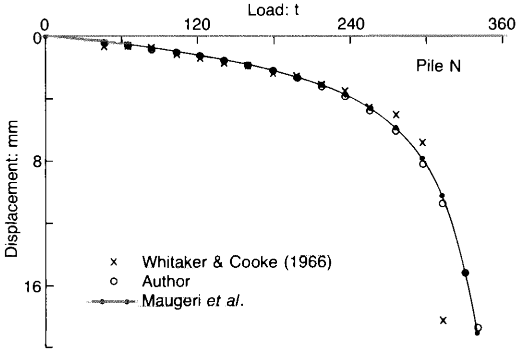

Fig. 19. Evaluation of total settlement for pile N

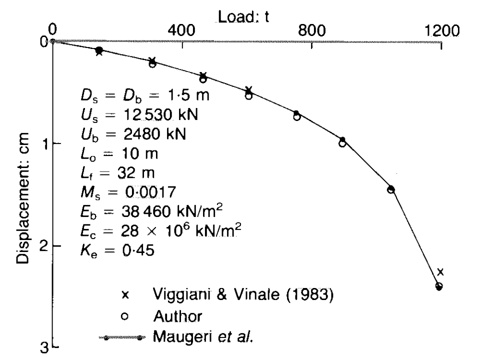

Fig. 20. Evaluation of total settlement for Viggiani & Vinale (1983) pile

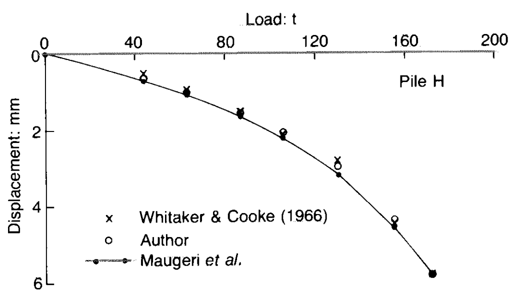

Fig. 18. Evaluation of total settlement for pile H

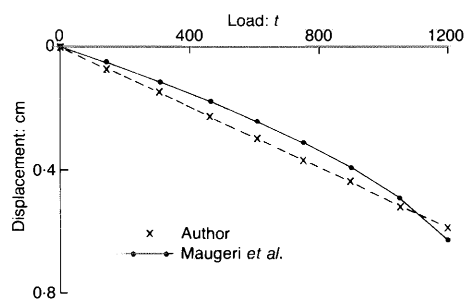

Fig. 21. Evaluation of elastic shortening Δ_e

lytical results will show good agreement with full-scale pile tests.

Author's reply

Since the Paper was written the routine use of the method for the analysis and prediction of single pile settlements and further research on it has been proceeding and the database has been widened to more than 500 case studies without any significant problems having arisen. Research has also been carried out on unloading recovery and it now appears that the maximum value of this can be predicted by a further extension of the same mathematical functions. This also shows how locked in stresses are produced after unloading in a pile. Other developments associated with the method have also been published (Fleming, 1992, 1993; England, 1992).

With regard to the violation of the single hyperbolic relationship when two hyperbolic functions are added for a given deformation, this is indeed inevitable. It is easy to demonstrate mathematically and largely explains the frequent ambivalence of engineers towards the plotting method. Tables 2–4 illustrate the problem. In Table 2 a hyperbolic shaft function characteristic for a rigid pile is shown. A similar characteristic for a pile base alone is shown in Table 3, and in Table 4 these are mathematically added together

Table 2. Rigid pile with $D_S = 1$ m, $U_S = 2000$ kN, $M_S = 0{\cdot}002$

Applied load: kN	Settlement: mm	Settlement/load $\times 10^3$	Interval slope: kN
181·81	0·2000	1·100	—
			2000
400	0·5000	1·250	—
			2000
620·69	0·9000	1·450	—
			2000
800	1·3333	1·6667	—
			2000
1000	2·0000	2·000	—
			2000
1200	3·0000	2·500	—
			2000
1428·58	5·0000	3·500	—
			2000
1600	8·0000	5·000	—
			2000
1800	18·0000	10·000	—

Table 3. Rigid pile with $D_B = 1$ m, $U_B = 1000$ kN, $E_B = 50\,000$ kN/m^2 (E_{25} base soil modulus)

Applied load: kN	Settlement: mm	Settlement/load $\times 10^3$	Interval slope: kN
16·39	0·2000	12·20	—
			1000
40	0·5000	12·50	—
			1000
69·77	0·9000	12·90	—
			1000
100	1·3333	13·33	—
			1000
142·86	2·0000	14·00	—
			1000
200	3·0000	15·00	—
			1000
294·12	5·0000	17·00	—
			1000
400	8·0000	20·00	—
			1000
600	18·0000	30·00	—

Table 4. Rigid pile with $D_S = 1$ m, $D_B = 1$ m, $U_S = 2000$ kN, $M_S = 0{\cdot}002$, $U_B = 1000$ kN, $E_B = 50\,000$ kN/m²

Applied load: kN	Settlement: mm	Settlement/load $\times 10^3$	Interval slope: kN
198·2	0·2000	1·00908	—
			2356·9
440	0·5000	1·13636	—
			2393·5
690·46	0·9000	1·303479	—
			2434·2
900	1·3333	1·4815	—
			2482·9
1142·86	2·0000	1·7500	—
			2545·2
1400	3·0000	2·1429	—
			2633·3
1722·7	5·0000	2·9024	—
			2733·2
2000	8·0000	4·0000	—
			2857·1
2400	18·0000	7 5000	—

for given settlements. The result, considering slope over the selected settlement intervals, clearly shows that the relationship Δ_T/P_T against Δ_T is no longer linear. It is undoubtedly true that in order to represent pile behaviour adequately, the functions representing the shaft and base, which are individually hyperbolic, have to be dealt with separately and subsequently combined to represent the whole pile. The method advocated by Chin (1970), for example, works well for piles which have nearly all their load carried either by shaft friction or end bearing, but is disappointing when these components act together and are nearly equal.

On the subject of quake in pile driving, where elastic shortening is restricted to that within the wave front and where volumetric strain along the shaft length is slight, it seems highly probable that quake Q is directly related to $M_S D_S$. Studies on base behaviour under impact seem to show that the stiffness E_B approaches a limiting value of the stiffness of water ($E_B = 2 \times 10^6$ kN/m²), which might not be entirely unexpected in fully saturated soils.

With regard to the question of the elastic shortening model, at an early stage other forms of analysis using the same basic functions were considered but it was decided to use the method in the Paper because it is straightforward, easily understood, and may be used as an everyday design and analysis tool. The elastic shortening model has since been refined for analysis purposes and the further suggestion of Messrs Maugeri, Castelli and Motta is welcomed. For design purposes, however, the more refined techniques make slight differences and are scarcely necessary, as Figs 18–21 imply. The model used for elastic shortening in the Paper actually produces a bilinear model which changes slope at the point where all shaft friction has been mobilized.

The values for M_S shown in the Paper are now borne out, at least for the stiffer ranges of soil, by a wealth of practical experience.

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